MODEL OF TURBULENCE FOR THE CALCULATION OF MIXED CONVECTION ALONG A VERTICAL WALL

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We consider mixed convection along a vertical wall using the k–ε model of turbulence. Expressions are obtained for the $\overline{u'v'}$ and $\overline{v't'}$ correlation functions and systematic calculations are carried out for turbulent mixed convection along a vertical wall with a constant heat flux across the wall.

Introduction. The k–ε model of turbulence developed by Jones and Launder is currently one of the most widely used models. Its application to convection above a hot horizontal surface in large-scale atmospheric boundary layers is well known [1-3]. In the present paper we consider a modification to the k–ε model for the case of mixed convection near a hot vertical plate.

Basic Equations. Following [1-3], we write the transport equations for $\overline{u_i'u_j'}$ and $\overline{u_j't'}$ in the form

$$\frac{D\overline{u_i'u_j'}}{Dt} = \mathrm{Diff}\,(\overline{u_i'u_j'}) + P_{ij} - \frac{2}{3}\delta_{ij}\varepsilon - C_1\frac{\varepsilon}{k}\times$$
$$\times\left(\overline{u_i'u_j'} - \frac{2}{3}\delta_{ij}k\right) - C_2\left(P_{ij} - \frac{2}{3}\delta_{ij}P\right), \tag{1}$$

$$\frac{D\overline{u_i't'}}{Dt} = \mathrm{Diff}\,(\overline{u_i't'}) - \overline{u_i'u_k'}\frac{\partial T}{\partial x_k} - C_{1T}\frac{\varepsilon}{k}\overline{u_i't'} + P_{iT}(1 - C_{2T}). \tag{2}$$

Here Diff is the diffusion transport operator; P_{ij} is the rate of production of $\overline{u_i'u_j'}$ resulting from the average velocity and buoyancy forces; P_{iT} is the rate of production of $\overline{u_i't'}$;

$$P \equiv -\left(\overline{u_i'u_k'}\frac{\partial U_i}{\partial x_k} + \beta g_i\overline{u_i't'}\right),$$

$$P_{ij} \equiv -\left(\overline{u_i'u_k'}\frac{\partial U_j}{\partial x_k} + \overline{u_j'u_k'}\frac{\partial U_i}{\partial x_k}\right) - \beta\,(g_i\overline{u_j't'} + g_j\overline{u_i't'}), \tag{3}$$

$$P_{iT} \equiv -\left(\overline{u_k't'}\frac{\partial U_i}{\partial x_k} + \beta g_i\overline{t'^2}\right).$$

In the first approximation, all transport processes in the boundary layer are assumed to occur in local equilibrium, i.e., the rate of growth of a fluctuation is equal to its rate of dissipation. Then it follows from (1) that

$$P_{ij} - \frac{2}{3}\delta_{ij}\varepsilon - C_1\frac{\varepsilon}{k}(\overline{u_i'u_j'} - \frac{2}{3}\delta_{ij}k) - C_2\left(P_{ij} - \frac{2}{3}\delta_{ij}P\right) = 0. \tag{4}$$

From the assumption that the boundary layer is in local equilibrium we have

$$\frac{2}{3}\delta_{ij}\varepsilon = \frac{2}{3}\delta_{ij}P. \tag{5}$$

Substituting (5) into (4), we obtain

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$$\frac{\overline{u_i' u_j'}-\frac{2}{3}\delta_{ij}k}{k}=\frac{1-C_2}{C_1}\,\frac{P_{ij}-\frac{2}{3}\delta_{ij}P}{\varepsilon}. \tag{6}$$

Similarly, we obtain from (2)

$$-\overline{u_i't'}=\frac{k}{\varepsilon}\,\frac{\overline{u_i'u_k'}\dfrac{\partial T}{\partial x_k}-(1-C_{2T})P_{iT}}{C_{1T}}=$$
$$=\frac{k}{\varepsilon}\,\frac{\overline{u_i'u_k'}\dfrac{\partial T}{\partial x_k}+(1-C_{2T})\left(\overline{u_k't'}\dfrac{\partial U_i}{\partial x_k}+g_i\beta\overline{t'^2}\right)}{C_{1T}}. \tag{7}$$

Since transport of the correlation function $\overline{t'^2}$ occurs in local equilibrium

$$P_t\equiv-2\overline{u_k't'}\frac{\partial T}{\partial x_k}=2\lambda\frac{\partial T}{\partial x_k}\frac{\partial T}{\partial x_k}\equiv\varepsilon_t. \tag{8}$$

Here P_t and ε_t are the rates of production and absorption of the quantity $\overline{t'^2}$ (see [3], for example).

Assuming that the time scales of the fluctuations $\overline{t'^2}$ and k are proportional, i.e.,

$$\overline{t'^2}/\varepsilon_t=0{,}5C_T' k/\varepsilon, \tag{9}$$

we obtain from (8) and (9)

$$\overline{t'^2}=-C_T'\frac{k}{\varepsilon}\overline{u_k't'}\frac{\partial T}{\partial x_k}. \tag{10}$$

Substituting (10) into (7), the equation for the correlation function $\overline{u_k't'}$ can be rewritten in the form

$$-\overline{u_i't'}=\frac{1}{C_{1T}}\frac{k}{\varepsilon}\left[\left(\overline{u_i'u_k'}\frac{\partial T}{\partial x_k}+(1-C_{2T})\times\right.\right.$$
$$\left.\left.\times\left(\overline{u_k't'}\frac{\partial U_i}{\partial x_k}-\beta g_iC_T'\frac{k}{\varepsilon}\overline{u_k't'}\frac{\partial T}{\partial x_k}\right)\right]. \tag{11}$$

For convection along a vertical wall ($g_1=-g$, $g_2=g_3=0$) we find from (6) and (10)

$$\overline{u_1'u_2'}=\frac{1-C_2}{C_1}\frac{k}{\varepsilon}P_{12}, \tag{12}$$

$$\overline{u_2't'}=-\frac{1}{C_{1T}}\left(\frac{k}{\varepsilon}\overline{u_2'u_k'}\frac{\partial T}{\partial x_k}+(1-C_{2T})\overline{u_k't'}\frac{\partial U_2}{\partial x_k}\right), \tag{13}$$

where

$$P_{12}=-\left(\overline{u_1'u_k'}\frac{\partial U_2}{\partial x_k}+\overline{u_2'u_k'}\frac{\partial U_1}{\partial x_k}\right)+\beta g\overline{u_2't'}. \tag{14}$$

Using boundary-layer theory, (13) and (14) can be simplified further

$$\overline{u_2't'}=-\frac{1}{C_{1T}}\frac{k}{\varepsilon}\overline{u_2'^2}\frac{\partial T}{\partial x_2}, \tag{15}$$

$$P_{12}=-\overline{u_2'^2}\frac{\partial U_1}{\partial x_2}+\beta g\overline{u_2't'}, \tag{16}$$

and hence we have

$$\overline{u_1'u_2'}=-\frac{1-C_2}{C_1}\frac{k}{\varepsilon}\overline{u_2'^2}\frac{\partial U_1}{\partial x_2}+\frac{1-C_2}{C_1}\frac{k}{\varepsilon}\beta g\overline{u_2't'}=$$
$$=U_{12}^0+\frac{1-C_2}{C_1}\frac{k}{\varepsilon}\beta g\overline{u_2't'}, \tag{17}$$

where

$$U_{12}^0 = -\frac{1-C_2}{C_1}\frac{k}{\varepsilon}\overline{u_2'^2}\frac{\partial U_1}{\partial x_2}. \tag{18}$$

Here U_{12}^0 is the value of the correlation function $\overline{u_1'u_2'}$ in the absence of buoyancy forces.

The result (18) does not take into account the effect of the wall. Hence we replaced (18) by the well-known formula of the k–ε hypothesis

$$U_{12}^0 = \overline{u_1'u_2'} = -C_\nu F_\nu \frac{k^2}{\varepsilon}\frac{\partial U_1}{\partial x_2}. \tag{19}$$

Comparing (18) and (15), we obtain an expression for $\overline{u_2't'}$

$$\overline{u_2't'} = \frac{C_{1T}C_1}{1-C_2} U_{12}^0 \frac{\partial T/\partial x_2}{\partial U_1/\partial x_2}. \tag{20}$$

Combining (17)-(20), we obtain equations for the turbulent transport coefficients

$$-\overline{u_1'u_2'} = \nu_t \frac{\partial U_1}{\partial x_2} = C_\nu F_\nu \frac{k^2}{\varepsilon}\left(\frac{\partial U_1}{\partial x_2} + \frac{C_0}{\mathrm{Pr}_{tf}}\frac{k}{\varepsilon}\beta g \frac{\partial T}{\partial x_2}\right), \tag{21}$$

$$-\overline{u_2't'} = a_t \frac{\partial T}{\partial x_2} = \frac{C_{1T}C_1}{1-C_2} C_\nu F_\nu \frac{k^2}{\varepsilon}\frac{\partial T}{\partial x_2} = \frac{1}{\mathrm{Pr}_{t\,f}} C_\nu F_\nu \frac{k^2}{\varepsilon}\frac{\partial T}{\partial x_2}. \tag{22}$$

Here Pr_{tf} is the value of Pr_t for forced convection and $C_0 = (1 - C_2)/C_1$.

We next consider (3), which determines the rate of production of turbulent kinetic energy. Assuming convection along a vertical wall and using boundary-layer theory, it can be written in the form

$$P = -\left(\overline{u_1'u_2'}\frac{\partial U_1}{\partial x_2} - \beta g \overline{u_1't'}\right). \tag{23}$$

In principle $\overline{u_1't'}$ can be expressed from (11) in the same way as was done for $\overline{u_2't'}$, but to avoid unwarranted complexity in the model we used the following relation obtained experimentally in [4] and supported by calculations in [2]:

$$\overline{u_1't'} = -2{,}1\overline{u_2't'}. \tag{24}$$

Combining (21)-(24) with the energy and momentum transport equations and the k–ε hypothesis, we obtain a system of equations for turbulent mixed convection along a vertical wall. To complete the computational model the generation term in the ε equation is written in a form which takes into account (23). The term taking into account the effect of the wall was multiplied by an additional damping factor, which limits the effect of this term to the viscous sublayer.

The final system of equations of the modified k–ε hypothesis is

$$U_1\frac{\partial k}{\partial x_1} + U_2\frac{\partial k}{\partial x_2} = \frac{\partial}{\partial x_2}\left(\nu + \frac{\nu_t}{CD_k}\right)\frac{\partial k}{\partial x_2} -$$

$$-\overline{u_1'u_2'}\frac{\partial U_1}{\partial x_2} - 2{,}1\beta g\overline{u_2't'} - \varepsilon - 2\nu\left(\frac{\partial \sqrt{k}}{\partial x_2}\right)^2,$$

$$U_1\frac{\partial \varepsilon}{\partial x_1} + U_2\frac{\partial \varepsilon}{\partial x_2} = \frac{\partial}{\partial x_2}\left(\nu + \frac{\nu_t}{CD_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_2} +$$

$$+ C_{\varepsilon 1}\frac{\varepsilon}{k}\left(-\overline{u_1'u_2'}\frac{\partial U_1}{\partial x_2} - 2{,}1\beta g\overline{u_2't'}\right) - C_{\varepsilon 2}F_\varepsilon\frac{\varepsilon^2}{k} - F_{\varepsilon 1}2\nu\nu_t\left(\frac{\partial^2 U_1}{\partial x_2^2}\right)^2.$$

The turbulent transport coefficients are given by (21) and (22). Here, $C_\nu = 0.09$, $CD_k = 1$, $CD_\varepsilon = 1.3$, $C_{\varepsilon 1} = 1.44$, $C_{\varepsilon 2} = 1.92$, $C_0 = 0.22$, $F_\nu = 1 - 0.3\exp(-\mathrm{Re}_t^2)$, $F_\varepsilon = \exp(-2.5/(1 + 0.02\,\mathrm{Re}_t))$,

$$F_{\varepsilon 1} = \begin{cases} 1, & y_+ \leqslant 3{,}5, \\ \exp(-(y_+/3{,}5 - 1)^2), & y_+ > 3{,}5. \end{cases}$$

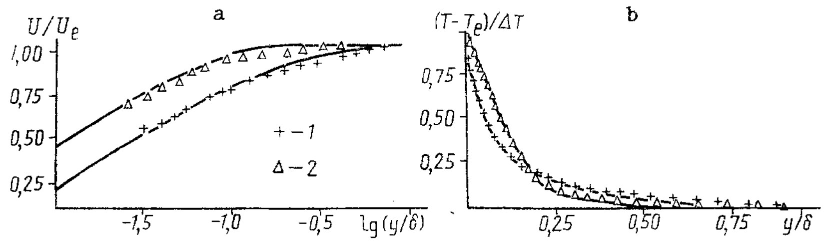

Fig. 1. Comparison of the calculated and experimental velocity and temperature profiles (Pr = 0.72). a: 1) Re = 2.96 × 10^5, Gr* = $5.33 \cdot 10^{13}$, 2) Re = $2.02 \cdot 10^5$, Gr* = $4.11 \cdot 10^{13}$; b: 1) Re = $5.0 \cdot 10^5$, $Gr^*/Re^{2.5}$ = 1.0, 2) Re = $5.0 \cdot 10^5$, $Gr^*/Re^{2.5}$ = 2.8.

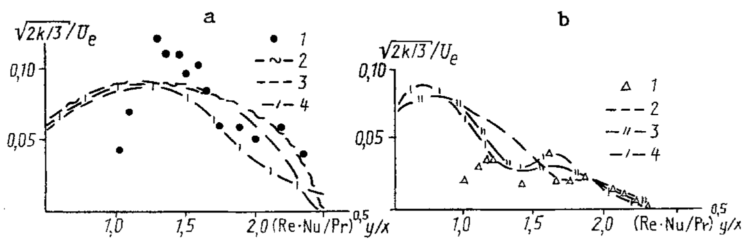

Fig. 2. Comparison of the calculated and experimental turbulent energy profiles (Pr = 0.72). a: 1) Re = $3.6 \cdot 10^5$, Gr* = $5.2 \cdot 10^{13}$, $Gr^*/Re^{2.5}$ = 0.67; 2) $3.45 \cdot 10^5$, $4.48 \cdot 10^{13}$, and 0.64; 3) $3.6 \cdot 10^5$, $5.21 \cdot 10^{13}$, and 0.67; 4) $2.81 \cdot 10^5$, $5.24 \cdot 10^{13}$, and 1.25; b: 1) Re = $2.8 \cdot 10^5$, Gr* = $5.2 \cdot 10^{13}$, $Gr^*/Re^{2.5}$ = 1.26; 2) $3.45 \cdot 10^5$, $2.24 \cdot 10^{14}$, and 3.20; 3) $3.45 \cdot 10^5$, $3.36 \cdot 10^{14}$, and 4.80; 4) $3.45 \cdot 10^5$, $4.48 \cdot 10^{14}$, and 6.41.

Standard values are used for the constants CD_ε, CD_k, $C_{\varepsilon 1}$, $C_{\varepsilon 2}$, C_ν and the functions F_ν and F_ε, and C_0 were calculated using the values of C_1 and C_2 given in [1]. The quantity Pr_{tf} was chosen from the best fit to the experimental data of the Nusselt number Nu calculated for forced convection on a given grid and with a given value of Pr. Here $1.1 \le Pr_{tf} \le 1.65$, $0.72 \le Pr \le 10$.

We assumed the boundary conditions

$$x_2 = 0, \quad k = \varepsilon = 0; \quad x_2 \to \infty, \quad k = 0, \quad \frac{\partial \varepsilon}{\partial x_2} = 0.$$

The calculations were started in the region where buoyancy forces are negligibly small. As approximate initial conditions we used the experimentally obtained temperature and velocity profiles for turbulent forced convection.

Calculated Results and Discussion. The computer program implementing our model was tested by comparing the calculated quantities with the experimental data.

The experiments known to the author on turbulent mixed convection along a vertical plate with a constant heat flux across the plate were carried out using a turbulator, but because of laminarization the turbulator did not have a significant effect on the temperature and velocity profiles. The calculated and experimental velocity and temperature profiles are shown in Fig. 1 (compare with [5]). Although the effect of the turbulator on the temperature and velocity was slight, the turbulator had a very strong effect on the fluctuations. Hence it is not possible to directly compare the calculated and experimental turbulent parameters. Several calculated profiles of $\sqrt{2k/3}/U_e$ are shown in Fig. 2. We see that the calculated curves are close to the experimental points and qualitatively reproduce the experimental behavior. The experimental data were taken from [5]. Unfortunately, these data are themselves widely scattered. Other experiments carried out under similar conditions are unknown to the author.

Figure 3 shows the calculated heat transfer as a function of Re and the parameter Gr^*/Re^4, which is independent of the longitudinal coordinate and characterizes the heating of

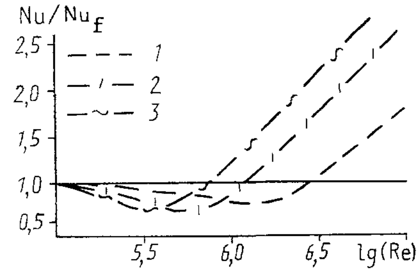

Fig. 3. Calculated heat transfer for mixed convection for different values of Gr^*/Re^4 ($Pr = 0.72$): 1) $Gr^*/Re^4 = 3.16 \cdot 10^{-9}$, 2) $1.58 \cdot 10^{-8}$, 3) $3.16 \cdot 10^{-8}$.

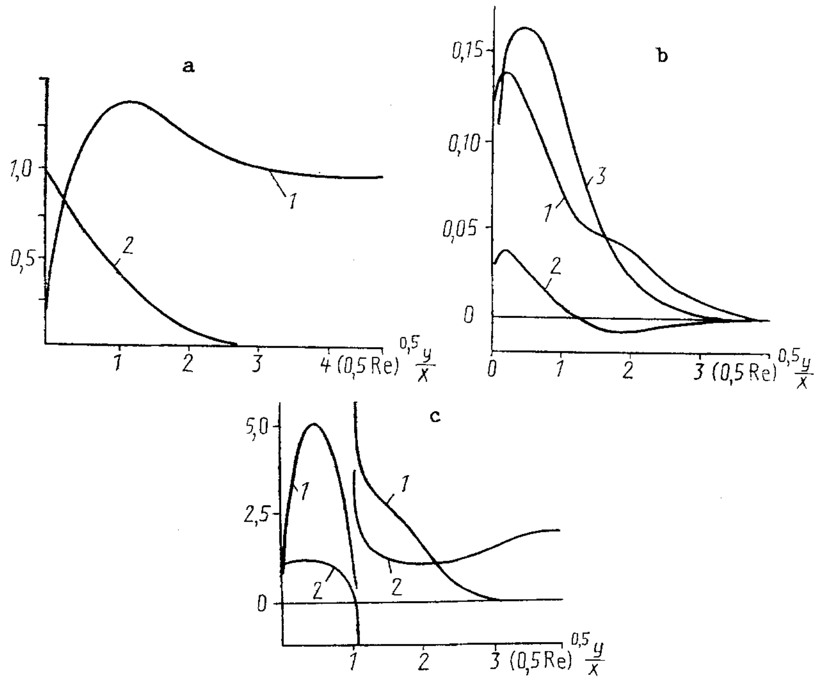

Fig. 4. Calculated flow profiles for turbulent mixed convection ($Re = 8.14 \cdot 10^5$, $Gr^* = 4.38 \cdot 10^{15}$, $Gr^*/Re^{2.5} = 7.32$, $Pr = 0.72$): a: 1) U_1/U_e, 2) $(T - T_e)/\Delta T$; b: 1) $(2k/3)^{0.5}/U_e$, 2) $10 \cdot (\overline{-u'v'}/U_e^2)$, 3) $0.1 \cdot (\overline{-v't'}/U_e \Delta T)$; c: 1) ν_t/ν, 2) Pr_t.

the wall. The figure shows the calculated quantity Nu/Nu_f. The calculations show that as Gr^*/Re^4 increases the quantity Nu/Nu_f decreases rapidly and its minimum value becomes smaller.

The calculated flow parameters for $Gr^*/Re^4 = 7.32$ are shown in Fig. 4. The velocity profile has a maximum. The turbulent energy shows the typical steplike behavior, which is supported by the experimental data of [6] for natural convection. The correlation function $\overline{u'v'}$ is negative in the outer part of the layer. There are unavoidable discontinuities in the ν_t/ν and Pr_t profiles. This is a result of the fact that the point where the correlation function $\overline{u'v'}$ vanishes is closer to the wall than the maximum in the velocity. The existence of a discontinuity follows from (21) and is supported by the data of [6].

It was necessary to remove these discontinuities artificially in doing the calculations. The calculated turbulent viscosity profile in mixed convection problems usually has to be corrected at one or two grid points, but this does not change the results markedly. However, it is impossible to avoid this problem in the k–ε model, which demonstrates the limited applicability of not only the k–ε model, but also the Boussinesq hypothesis on the turbulent viscosity for flow along a vertical wall in the presence of buoyancy forces.

Conclusions. The modification of the k–ε model given here is free of additional equations and new empirical constants and can be used to take into account the effect of buoyancy forces on the formation of a turbulent boundary layer along a vertical wall. The computer program written by the author on the basis of the modified model produced results which are in satisfactory agreement with experiment. The calculated results for the boundary layer demonstrate the limited applicability of the Boussinesq hypothesis on the turbulent viscosity for flow in the presence of buoyancy forces.

NOTATION

$k = 0.5\overline{u_i'u_j'}$, turbulent energy; ε, rate of dissipation of turbulent energy; U_i, average velocity in the i-th direction; u_i', fluctuation component of the velocity in the i-th direction; U_e, velocity of the approach stream; T, average temperature; t', fluctuation component of the temperature; $Re = U_e x/\nu$, Reynolds number; $Re_t = \sqrt{kL}/\nu$, turbulent Reynolds number; $Gr^* = g\beta q_w x^4/\lambda\nu^2$, modified Grashof number; $Nu = \alpha x/\lambda$, Nusselt number; Nu_f, Nusselt number for forced flow; $Pr = \nu/a$, Prandtl number; $Pr_t = \nu_t/a_t$, turbulent Prandtl number; Pr_{tf}, turbulent Prandtl number for forced flow; δ_{ij}, Kronecker–Kapelli delta.

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EXPERIMENTAL INVESTIGATION OF HEAT TRANSFER AND HYDRODYNAMICS IN STEADY AND PULSATING LAMINAR FLOW OF A NONLINEARLY VISCOUS FLUID IN A TUBE WITH A HELICAL TAPE INSERT

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The steady and pulsating laminar flow of a non-Newtonian fluid in a helical duct is investigated experimentally.

The laminar flow of viscous fluids is frequently encountered in equipment used in the chemical and petrochemical industries. Heat transfer is limited in such equipment by the low heat-transfer coefficients of viscous fluids flowing in smooth tubes. A well-known and simple technique for the augmentation of convective heat transfer in tubes is the application of twisted-tape inserts. Exhaustive experimental data have now been accumulated on heat transfer in tubes with tape augmenters [1-5]. However, in the study of heat-transfer augmentation methods it is useful to investigate the combined effects of several heat-transfer augmentation techniques.

Several authors (e.g., Fedotkin and Firisyuk [6]) have investigated and analyzed a number of possible combinations of augmentation methods, mainly of a design nature, and then in application to viscous Newtonian fluids. Considerable attention has been given to the study of phenomena associated with the unsteady (fluctuating) flow of non-Newtonian fluids and their influence on heat transfer and hydraulic friction [7, 8].

The objective of the present study is to investigate the influence of the superposition of pulsations onto a nonlinearly viscous fluid flow on heat transfer and hydraulic friction

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